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The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. To better understand the science of propulsion it is necessary to use some mathematical ideas from vector analysis. Most people are introduced to vectors in high school or college, but for the elementary and middle school students, or the mathematically-challenged: DON'T PANIC!. There are many complex parts to vector analysis and we aren't going there. We are going to limit ourselves to the very basics. Vectors allow us to look at complex, multi-dimensional problems as a simpler group of one-dimensional problems. We will be concerned mostly with definitions The words are a bit strange, but the ideas are very powerful as you will see. If you want to find out a lot more about vectors you can download this report on vector analysis. Math and science were invented by humans to describe and understand the world around us. We live in a (at least) four-dimensional world governed by the passing of time and three space dimensions; up and down, left and right, and back and forth. We observe that there are some quantities and processes in our world that depend on the direction in which they occur, and there are some quantities that do not depend on direction. For example, the volume of an object, the three-dimensional space that an object occupies, does not depend on direction. If we have a 5 cubic foot block of iron and we move it up and down and then left and right, we still have a 5 cubic foot block of iron. On the other hand, the location, of an object does depend on direction. If we move the 5 cubic foot block 5 miles to the north, the resulting location is very different than if we moved it 5 miles to the east. Mathematicians and scientists call a quantity which depends on direction a vector quantity. A quantity which does not depend on direction is called a scalar quantity. Vector quantities have two characteristics, a magnitude and a direction. Scalar quantities have only a magnitude. When comparing two vector quantities of the same type, you have to compare both the magnitude and the direction. For scalars, you only have to compare the magnitude. When doing any mathematical operation on a vector quantity (like adding, subtracting, multiplying ..) you have to consider both the magnitude and the direction. This makes dealing with vector quantities a little more complicated than scalars. On the slide we list some of the physical quantities discussed in the Beginner's Guide to Aeronautics and group them into either vector or scalar quantities. Of particular interest, the forces which operate on a flying aircraft, the weight, thrust, and aerodynamic forces, are all vector quantities. The resulting motion of the aircraft in terms of displacement, velocity, and acceleration are also vector quantities. These quantities can be determined by application of Newton's laws for vectors. The scalar quantities include most of the thermodynamic state variables involved with the propulsion system, such as the density, pressure, and temperature of the propellants. The energy, work, and entropy associated with the engines are also scalar quantities. Vectors have magnitude and direction, scalars only have magnitude. The fact that magnitude occurs for both scalars and vectors can lead to some confusion. There are some quantities, like speed, which have very special definitions for scientists. By definition, speed is the scalar magnitude of a velocity vector. A car going down the road has a speed of 50 mph. Its velocity is 50 mph in the northeast direction. It can get very confusing when the terms are used interchangeably! Another example is mass and weight. Weight is a force which is a vector and has a magnitude and direction. Mass is a scalar. Weight and mass are related to one another, but they are not the same quantity.` While Newton's laws describe the resulting motion of a solid, there are special equations which describe the motion of fluids, gases and liquids. For any physical system, the mass, momentum, and energy of the system must be conserved. Mass and energy are scalar quantities, while momentum is a vector quantity. This results in a coupled set of equations, called the Navier-Stokes equations, which describe how fluids behave when subjected to external forces. These equations are the fluid equivalent of Newton's laws of motion and are very difficult to solve and understand. A simplified version of the equations called the Euler equations can be solved for some fluids problems. Activities: Guided Tours Navigation .. Beginner's Guide Home Page Scalars and vectors are fundamental concepts in physics that describe different types of quantities. Scalars are quantities that are fully described by a magnitude (or numerical value) alone, such as temperature, mass, speed, and time. They have no direction associated with them. In contrast, vectors are quantities that have both magnitude and direction. Examples of vectors include displacement, velocity, acceleration, and force. Understanding the distinction between scalars and vectors is crucial for solving problems in physics, including vector algebra and determining the units of momentum and units of velocity, as vectors require consideration of both their magnitude and direction for accurate representation and calculation. Scalars and vectors are fundamental concepts in physics that describe the properties of quantities. Scalars are quantities that only have magnitude, such as temperature, mass, or speed. They are completely described by a single numerical value and a unit. Vectors, however, have both magnitude and direction, making them more complex. Examples of vectors include velocity, force, and displacement. Scalars are quantities that only have magnitude. They are described by a single value with units. Speed (s): s = d/t Speed is the rate at which an object covers distance. Here, ddd represents the total distance traveled, and ttt is the time taken. Speed is a scalar because it does not include any directional information. Energy (E): E=mc2 In Einstein's mass-energy equivalence formula, EEE represents energy, mmm is mass, and ccc is the speed of light in a vacuum (approximately 3×1083 \times 10^83×108 meters per second). This formula shows that mass can be converted into energy. Energy is a scalar quantity. Vectors have both magnitude and direction. They are represented by arrows in diagrams, where the length of the arrow indicates the magnitude and the direction of the arrow shows the direction of the vector. Displacement (d⃗): d = r2−r1 Displacement is the vector that shows the change in position of an object. r1 is the initial position vector, r2and is the final position vector. Displacement considers direction, making it a vector. Velocity (v⃗): v = d/t Velocity is the rate of change of displacement with time. Unlike speed, velocity is a vector because it includes direction. Here,d is displacement and t is time. Force (F⃗): F = ma According to Newton's second law of motion, force is the product of mass and acceleration. Here, m is mass and a is acceleration. Force is a vector because it causes an object to move in a specific direction. Temperature: Temperature is a scalar quantity that measures the average kinetic energy of particles in a substance. It is fully described by its magnitude, 25°C, without any directional component. Mass : Mass represents the amount of matter in an object. It is a scalar quantity described by a single value, 10 kilograms, and does not depend on direction. Speed : Speed is the rate at which an object covers distance. As a scalar, it only has magnitude, such as 60 km/h, and no directional information. Distance : Distance is a scalar that measures the total path length traveled by an object. For example, 5 meters indicates how far an object has moved, without specifying direction. Energy : Energy quantifies the capacity to do work. It is a scalar quantity; 100 Joules of energy can be transferred or transformed without a direction. Time : Time is a scalar representing the duration of an event. Three seconds indicate the length of time elapsed, independent of direction. Volume : Volume measures the amount of space occupied by an object. It is a scalar quantity, such as 2 liters, with only magnitude. Density : Density is the mass per unit volume of a substance. As a scalar, it is described by its magnitude, 1.2 g/cm³, without direction. Pressure : Pressure is the force exerted per unit area. It is a scalar quantity; 101.3 kPa indicates the magnitude of pressure applied uniformly in all directions. Work : Work measures the energy transferred by a force over a distance. It is a scalar quantity, with 500 Joules representing the magnitude of work done. Power : Power is the rate at which work is done or energy is transferred. As a scalar, 60 Watts quantifies the magnitude of power without direction. Charge : Charge represents the quantity of electricity. It is a scalar quantity, such as 1.6 x 10^{-19} Coulombs, with only magnitude. Luminous Intensity : Luminous intensity measures the brightness of a light source. It is a scalar quantity, with 100 Candela indicating its magnitude. Electric Potential : Electric potential represents the electric potential energy per unit charge. As a scalar, 12 Volts describes its magnitude without direction. Frequency : Frequency measures the number of cycles per second of a periodic event. It is a scalar quantity; 50 Hertz indicates its magnitude. Velocity : Velocity is a vector quantity that includes both speed and direction. For example, 50 km/h north specifies both the magnitude of speed and the direction of motion. Force : Force is a vector quantity characterized by its magnitude and direction. Twenty Newtons upwards indicates the strength and direction of the force applied. Displacement : Displacement measures the change in position of an object. It is a vector quantity, with 10 meters east specifying both the distance and direction from the starting point. Acceleration : Acceleration is a vector quantity that describes the rate of change of velocity. For example, 9.8 m/s² downwards indicates both the magnitude and the direction of acceleration due to gravity. Momentum : Momentum is a vector quantity that combines mass and velocity. Fifteen kg·m/s at 30° to the horizontal indicates its magnitude and direction. Electric Field : The electric field is a vector that represents the force per unit charge. For instance, 100 N/C to the right specifies both the strength and direction of the electric field. Magnetic Field : The magnetic field is a vector quantity that describes the magnetic influence. For example, 0.5 Tesla upwards indicates the magnitude and direction of the field. Torque : Torque is a vector quantity representing the rotational force. Thirty Newton-meters clockwise specifies its magnitude and rotational direction. Gravitational Field : The gravitational field is a vector that measures the gravitational force per unit mass. For example, 9.8 N/kg downwards indicates the strength and direction of gravity. Impulse : Impulse is a vector quantity representing the change in momentum. Two hundred N·s north specifies both the magnitude and the direction of the applied force. Current Density : Current density is a vector that describes the electric current per unit area. For example, 3 A/m² to the right indicates both the magnitude and direction. Angular Momentum : Angular momentum is a vector quantity describing rotational motion. Five kg·m²/s counterclockwise specifies its magnitude and direction. Linear Momentum : Linear momentum is a vector that combines mass and velocity. For instance, 25 kg·m/s west indicates its magnitude and direction. Stress : Stress is a vector quantity representing force per unit area within materials. Forty N/m² upwards indicates the magnitude and direction of the internal force. Position : Position is a vector that specifies the location of an object. For example, 3 meters southeast indicates both the distance and direction from a reference point. Scalar quantities are physical quantities that have only magnitude and no direction. They are completely described by a single numerical value and a unit. Common examples of scalar quantities include time, energy, distance, volume, and density. Scalars are straightforward because they do not involve any directional component, making them easier to handle in calculations and everyday measurements. Vector quantities, on the other hand, have both magnitude and direction, adding a layer of complexity to their description. Vectors are represented by arrows, where the length of the arrow indicates the magnitude and the arrowhead points in the direction. Examples of vector quantities include velocity, which specifies the speed of an object and the direction it is moving; force, which describes the push or pull on an object in a specific direction; and displacement, which indicates the change in position of an object from one point to another. Other vector quantities include acceleration, momentum, electric field, magnetic field, and weight. Understanding vector quantities is essential in physics, as they provide a more comprehensive description of many physical phenomena, requiring vector addition and subtraction for accurate analysis. AspectScalarVectorDefinitionA quantity with only magnitude.A quantity with both magnitude and direction.ExamplesTemperature, mass, speed, time, energyVelocity, force, displacement, accelerationRepresentedRepresented by a single number and unit.Represented by an arrow with length (magnitude) and direction.AdditionAdded using simple arithmetic.Added using vector addition rules (parallelogram law or triangle method).SubtractionSubtracted using simple arithmetic.Subtracted using vector subtraction rules.MultiplicationMultiplied or divided by other scalars directly.Multiplied by scalars to change magnitude, but direction remains the same.Examples in PhysicsEnergy (Joules), distance (meters)Force (Newtons), velocity (meters per second) Scalars and vectors each have distinct properties that are essential for understanding and solving physics problems. Magnitude Only: Scalars are described by a single numerical value, representing the magnitude. No Direction: Scalars do not have any directional component. Examples: Common scalar quantities include mass, temperature, speed, and time. Addition/Subtraction: Scalars can be added or subtracted using simple arithmetic. Units: Scalars have units that correspond to their physical quantity, such as kilograms for mass or seconds for time. Magnitude and Direction: Vectors are described by both a magnitude and a direction. Representation: Vectors are often represented graphically by arrows, where the length represents the magnitude and the arrowhead indicates the direction. Examples: Common vector quantities include displacement, velocity, acceleration, and force. Vector Algebra: Vectors follow specific rules for addition, subtraction, and multiplication, including the use of vector components and unit vectors. Units of Momentum and Velocity: Vectors like momentum and velocity have specific units, such as kg·m/s for momentum and m/s for velocity. Operations: Vector operations include the dot product and cross product, which are used in physics to calculate work, torque, and other quantities. A vector is represented by an arrow. The length represents the magnitude, and the arrow points in the direction. Speed is a scalar quantity, indicating how fast an object moves. Velocity is a vector, indicating speed with direction. Vectors are added using the head-to-tail method or by breaking them into components and adding corresponding components. The result is a vector whose magnitude is scaled by the scalar and direction remains unchanged. A unit vector has a magnitude of one and indicates direction only. It is used to specify direction in vector operations. No, a vector cannot have a negative magnitude, but its direction can be reversed. The magnitude of a vector is found using the Pythagorean theorem for its components. Distance is a scalar quantity, measuring the path length. Displacement is a vector, measuring the straight-line distance between initial and final points. Vectors are used for quantities like force, acceleration, and velocity, which have both magnitude and direction. Yes, vector subtraction is performed by adding the negative of the vector to be subtracted, using the head-to-tail method. Tone Friendly Formal Casual Instructive Professional Empathetic Humorous Serious Optimistic Neutral 10 Examples of Public speaking 20 Examples of Gas lighting Which of the following is a scalar quantity? Choose the correct answer A quantity with magnitude only A quantity with both magnitude and direction A quantity that is always positive A quantity that cannot be measured Which of the following represents a vector quantity? Choose the correct answer When two vectors are added together, what is the resulting vector called? Choose the correct answer Which of the following operations is not valid for vectors? Choose the correct answer Multiplication by a scalar In vector addition, what method can be used to find the resultant vector if two vectors are perpendicular to each other? Choose the correct answer If a vector A has a magnitude of 5 units and is directed 30 degrees above the horizontal, what is the horizontal component of the vector? Choose the correct answer Which of the following statements about scalar and vector quantities is true? Choose the correct answer Scalars can be added only to other scalars Vectors cannot be multiplied by scalars Vectors are always positive Scalars can be subtracted from other vectors Which operation can be performed on vectors to determine the angle between them? Choose the correct answer What is the result of the vector subtraction A - B if A and B are vectors of equal magnitude and opposite directions? Choose the correct answer A vector with double the magnitude of A or B A scalar quantity A zero vector A vector in the direction of A Before you leave, take this Scalar and Vector Quiz Start QuizIn mathematics and physics, a scalar is a quantity that only has magnitude (size), while a vector has both magnitude and direction. Examples of scalar quantities include pure numbers, mass, speed, temperature, energy, volume, and time. Examples of vector quantities include velocity, acceleration, momentum, displacement, and forces, such as weight and friction.ScalarVectorIndicates magnitude or sizeIndicates magnitude and directionExamples include mass, temperature, speed, volume, and timeExamples include velocity, acceleration, weight, and frictionMay or may not include units (e.g., meters, kilograms)May be written in bold text or with an arrow over it to distinguish it from a scalar. Direction may be positive/negative or other (e.g., east, left, down).Mathematical operations involving scalars produce another scalar. Operations on a scalar and a vector produce a vector.Operations involving vectors produce a vector. The dot product of two vectors is a scalar. Summation, subtraction, or the cross product of two vectors is a vector.Here are some examples of scalar quantities:MassSpeedLengthVolumeDensityPowerPressureTemperatureHere are some examples of vector quantities:ForceWeightFrictionAccelerationMomentum(1) The car is going 75 mph.This is a scalar value because you don't know which direction the car is going.(2) You walk 4 mph toward the store.This is a vector because you know the speed and direction.(3) The box in the west corner of the room has a mass of 12 kilograms.The mass of the box is a scalar quantity. Even though you know the location of the box, this fact has nothing to do with its mass.(5) The time is 12:30 pm.This is a scalar. There is no direction.(6) The pressure inside a balloon is 2 atmospheres.Pressure has a magnitude, but it does not have a direction. Another way of looking at it is that pressure acts in all directions at once.(7) The cat weighs 8 pounds.Mass and weight can be confusing, when it comes to distinguishing between scalar and vector quantities. The pound is a unit of weight, so this value is a vector. The implied direction is down, toward the Earth's center of gravity. If the "weight" of the cat was given in kilograms, it would be a scalar value (for mass, not weight). A cat's weight is different on the Moon or Mars, but its mass remains the same.A unit vector is a vector that has magnitude equal to one. It's usually indicated by placing a caret (^) over it. The unit vector x, with a carat over it, is read as "x-hat" because the vector looks somewhat like it's wearing a hat.The null vector or zero vector is a vector with a magnitude of zero. While it has no magnitude, it has a direction. For example, you could use a null vector to indicate "down" without specifying a particular value. The direction indicated by a compass is pointing.Ashcroft, Neil; Mermin, N. David (1976). Solid State Physics. Toronto: Thomson Learning. ISBN 978-0-03-083993-1.Banach, Stefan (1922). "Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales" (On operations in abstract sets and their application to integral equations)". Fundamenta Mathematicae (in French). 3: 133–181. doi:10.4064/fm-3-1-133-181Lay, David C. (2006). Linear Algebra and Its Applications (3rd ed.). Addison–Wesley. ISBN 0-321-28713-4.Strang, Gilbert (2006). Linear Algebra and Its Applications (4th ed.). Brooks Cole. ISBN 0-03-010567-6.Related Posts Vectors and scalars are two ways in which we can express physical quantities. The use of each depends on the nature of each quantity. There are physical quantities that can be defined entirely by a single number and unit. Other physical quantities have a direction and cannot be expressed by a single number. On the other hand, a scalar quantity is a physical quantity that can be expressed by a single number. An illustration of this is the motion of a car: we must specify not only how fast it is moving, but we must also consider the direction in which it is moving. Since vectors and scalars have different properties, we usually use letters with arrows, like $latex \vec{A}$, to indicate that vectors have direction. Main characteristics of scalars Scalar quantities are represented by a single number (magnitude) and a unit. Ordinary arithmetic operations apply to scalar quantities. For example, 3 km + 5 km = 8 km. Scalars are usually represented by common letters, such as A, T, S. Main characteristics of vectors Vectors are defined by a magnitude and a direction. Combining vectors requires a different set of operations than ordinary arithmetic. Vectors are usually represented by a letter with an arrow above it. For example, $latex \vec{A}, ~\vec{B}$. Two vectors are parallel if they have the same direction. Two vectors are equal only if they have the same magnitude and the same direction. The magnitude of a vector is a scalar quantity. The following are some examples of vector quantities and scalar quantities: ScalarsVectorsDistanceDisplacementTimeVelocityTemperatureAccelerationMassForceDensityMomentumEnergy We can understand the difference between vectors and scalars by considering distance (a scalar quantity) and displacement (a vector quantity). Displacement is the change in position of an object. Displacement is a vector quantity because we must specify the direction we have moved. For example, walking 500 m north will not take us to the same place as walking 500 m south. These two displacements have the same magnitude, but different directions. Note that displacement is not directly related to the total distance traveled. We can represent the displacement by an arrow pointing in the direction of displacement: The displacement is always a straight line from the starting point to the end point, i.e. it does not depend on the path taken, even if it is curved: If an object goes to point P2 and returns to point P1, the displacement is 0. That is, the total displacement for a round trip is 0, regardless of the distance traveled: You have completed the quiz! Interested in learning more about vectors? Check out these pages: .author-box {margin: 70px 0; padding: 30px; background-color: #f9fcff; border-radius: 15px; box-shadow: 0px 0px 5px 0px #ccc; max-width:1100px; margin-left:auto !important; margin-right:0px !important; } .author-box img {margin:auto; border-radius: 50%;} .author-box h3 {margin-top: 20px; font-size:19px;} .author-box p {margin: 10px 0; text-align:left; } .author-box a {display: inline-block; margin-right: 10px; color: black; text-decoration: none;} { "@context": " ", "@type": "Person", "name": "Jefferson Huera Guzman", "image": " ", "url": " ", "description": "Jefferson is the lead author and administrator of Neurochispas.com.", "sameAs": [" ", " ", " "], "email": "", "worksFor": { "@type": "Organization", "name": "Interacti Digital LLC"}, "alumniOf": { "@type": "CollegeOrUniversity", "name": "The University of Manchester"}, "knowsAbout": ["Algebra", "Calculus", "Geometry", "Mathematics", "Physics"] } Scalar and Vector Quantities are an important concept in physics. Scalar Quantities are defined as physical quantities that have magnitude or size only. For example, distance, speed, mass, density, etc.However, vector quantities are those physical quantities that have both magnitude and direction like displacement, velocity, acceleration, force, etc. It should be noted that when a vector quantity changes its magnitude and direction also changes similarly, when a scalar quantity changes, only its magnitude changes.Scalar Quantities DefinitionA scalar quantity is a physical quantity that has only magnitude and no direction.In other words, a scalar quantity is described only by a number and a unit, and it does not have any associated direction or components. Examples of scalar quantities include temperature, mass, time, distance, speed, and energy. These quantities can be measured using instruments such as thermometers, scales, stopwatches, rulers, speedometers, and wattmeters.Other than these some more scalars are:AreaVolumeDensityTemperatureElectric ChargeGravitational ForceScalar quantities can be added, subtracted, multiplied, and divided using standard mathematical operations. For example, if a car travels 100 kilometers in 2 hours, its average speed can be calculated as 50 kilometers per hour (km/h) by dividing the distance traveled by the time taken.Scalar quantities are often contrasted with vector quantities, which have both magnitude and direction, such as velocity, acceleration, force, and displacement. Vector quantities are typically represented graphically using arrows to show their direction and magnitude, while scalar quantities are represented using only a number and a unit.Scalar QuantitiesA vector quantity is a physical quantity that has both magnitude and direction.In other words, a vector quantity is described by a number, a unit, and a direction.For example, if a car is traveling at a velocity of 50 km/h towards the east, its velocity can be represented as a vector with an arrow pointing to the right (east) and a length of 50 km/h.There are countless examples of vector quantities in daily life. The list of some of them is shown below!ForcePressureThrustElectric FieldPolarizationWeightVector QuantitiesExamples of vector quantities include velocity, acceleration, force, displacement, and momentum. These quantities are commonly represented graphically using arrows to show both their direction and magnitude. Vector quantities can be added, subtracted, multiplied, and divided using vector algebra. For example, if a force of 10 N is applied to an object in the north direction, and a force of 5 N is applied in the east direction, the resultant force can be calculated using vector addition as a force of √125 N towards the northeast direction.Vector quantities are used in many fields of science and engineering, such as mechanics, electromagnetism, fluid dynamics, and quantum mechanics. They are essential for describing the behavior of physical systems and making predictions about their future states.Vector NotationVector notation is a way or notation used to represent a quantity that is a vector, through an arrow (→) above its symbol, as shown below:Scalar and Vector QuantityThe differences between Scalar and Vector Quantities are shown in the table added below,Difference Between Scalar and Vector QuantityScalarVectorScalar quantities have magnitude or size only.Vector quantities have both magnitude and direction.It is known that every scalar exists in one dimension only. Vector quantities can exist in one, two, or three-dimension.Whenever there is a change in a scalar quantity, can correspond to a change in its magnitude also.Any change in a vector quantity can correspond to cha change in either its magnitude or direction or both.These quantities can not be resolved into their components.These quantities can be resolved into their components, using the sine or cosine of the adjacent angle.Any mathematical process that involves more than two scalar quantities will only give scalars.Mathematical operations on two or more vectors can provide either a scalar or a vector as a result. For instance, the dot product of two vectors only produces a scalar, whereas the cross product, sum, or subtraction of two vectors gives a vector.Some examples of Scalar quantities are:MassSpeedDistanceTimeAreaVolumeSome examples of Vector quantities are:VelocityForcePressureDisplacementAccelerationEquality of VectorsTwo vectors are considered to be equal when they have the same magnitude and same direction. The figure below shows two vectors that are equal, notice that vectors are parallel to each other and have the same length. The second part of the figure shows two unequal vectors, which even though have the same magnitude, are not equal because they have different directions.Multiplication of Vectors with ScalarMultiplying a vector with a constant scalar k gives a vector whose direction is the same but the magnitude is changed by a factor of k. The figure below shows the vector before and after it is multiplied by the constant k. In mathematical terms, this can be rewritten as, |k\vec{v}| = k|\vec{v}| if k > 1, the magnitude of the vector increase while it decreases when the k < 1. Addition of VectorsVectors cannot be added by usual algebraic rules. While adding two vectors, the magnitude and the direction of the vectors must be taken into account.Triangle law is used to add two vectors, the diagram below shows two vectors "a" and "b" and the resultant is calculated after their addition. Vector addition follows commutative property, this means that the resultant vector is independent of the order in which the two vectors are added. \vec{a} + \vec{b} = \vec{c} \vec{a} + \vec{b} + \vec{b} + \vec{a} - (Commutative Property)Triangle Law of Vector AdditionConsider the vectors given in the figure above. The line PQ represents the vector "p", and QR represents the vector "q". The line QR represents the resultant vector. The direction of AC is from A to C. Line AC represents, \vec{p} + \vec{q} The magnitude of the resultant vector is given by, \sqrt{|p|^2 + |q|^2 + 2|p||q|cos(\theta)} θ represents the angle between the two vectors. Let φ be the angle made by the resultant vector with the vector p.tan (\phi) = \dfrac{q\sin\theta}{p + q\cos\theta} The above formula is known as the Triangle Law of Vector Addition.Parallelogram Law of Vector AdditionThis law is just another way of understanding vector addition. This law states that if two vectors acting on the same point are represented by the sides of the parallelogram, then the resultant vector of these vectors is represented by the diagonals of the parallelograms.The figure below shows these two vectors represented on the sides of the parallelogram. Also, Check:Vector AlgebraDot and Cross Product of VectorsExamples on Scalar and VectorExample 1: Find the magnitude of the vector, v = i + 4j. Solution: |v| = \sqrt{a^2 + b^2} a = 1, b = 4|v| = \sqrt{1^2 + 4^2} |v| = √17Example 2: A vectorplar is given by, v = i + 4j. Find the magnitude of the vector when it is scaled by a constant of 5. Solution: |v| = \sqrt{a^2 + b^2} 5|v| = |5v| a = 1, b = 4 5|v| = |5(i + 4j)| |5i + 20j| |v| = \sqrt{5^2 + 20^2} |v| = \sqrt{25 + 400} |v| = √425Example 3: A vector is given by, v = i + j. Find the magnitude of the vector when it is scaled by a constant of 0.5. Solution: |v| = \sqrt{a^2 + b^2} 0.5|v| = |0.5v| a = 1, b = 1|0.5v| = |0.5(i + j)| |0.5i + 0.5j| |v| = \sqrt{0.5^2 + 0.5^2} |v| = \sqrt{0.25 + 0.25} |v| = √0.5Example 4: Two vectors with magnitude 3 and 4. These vectors have a 90° angle between them. Find the magnitude of the resultant vectors. Solution: Let the two vectors be given by p and q. Then resultant vector "r" is given by, |r| = \sqrt{|p|^2 + |q|^2 + 2|p||q|cos(\theta)} |p| = 3, |q| = 4 and \theta = 90^o |r| = \sqrt{|p|^2 + |q|^2 + 2|p||q|cos(\theta)} |r| = \sqrt{|3|^2 + |4|^2 + 2|3||4|cos(90)} |r| = \sqrt{|3|^2 + |4|^2} |r| = \sqrt{9 + 16} |r| = 5Example 5: Two vectors with magnitude 10 and 9. These vectors have a 60° angle between them. Find the magnitude of the resultant vectors. Solution: Let the two vectors be given by p and q. Then resultant vector "r" is given by, |r| = \sqrt{|p|^2 + |q|^2 + 2|p||q|cos(\theta)} |p| = 10, |q| = 9 and \theta = 60^o |r| = \sqrt{|p|^2 + |q|^2 + 2|p||q|cos(\theta)} |r| = \sqrt{|10|^2 + |9|^2 + 2|10||9|cos(60)} |r| = \sqrt{100 + 81 + 90} |r| = \sqrt{271} Physics is the science that seeks to describe the natural world with precision. Whether you're studying the motion of planets or the flow of electricity through a wire, physics provides the language and the mathematics to interpret and predict behavior in the universe. But before we can dive into advanced theories or equations, we must understand some of the most fundamental concepts that form the backbone of this discipline. Two such foundational ideas are scalars and vectors. These terms might sound like dry definitions from a textbook, but in reality, they are essential to how we interpret everything from throwing a ball to launching a rocket. They define how quantities behave, how they are manipulated mathematically, and how they interact with one another. Grasping the distinction between a scalar and a vector is not just academic—it's a doorway into the deeper beauty of physics itself.In this article, we will explore the differences between scalars and vectors, dive into how they manifest in the physical world, examine their mathematical representations, and discuss why these differences matter. Prepare for a deep dive into one of physics' most vital distinctions—an exploration that might change how you look at motion, energy, and even reality itself.Let us begin with the simpler of the two: the scalar. A scalar is a quantity that has magnitude only. In layman's terms, it's something you can measure with a single number and unit. Scalars do not care about direction. They are concerned solely with "how much" or "how big."Temperature is a scalar. If you say it's 30 degrees Celsius, you've provided a complete piece of information. There's no direction associated with temperature; it's just a measure of thermal energy.Speed is another scalar. If a car is moving at 60 kilometers per hour, we know how fast it's going, but not where. Volume, mass, energy, and time are all scalars, too. Each of these can be completely described by a single numerical value.Despite their simplicity, scalars are indispensable. They form the quantities you add up, compare, or plug into equations in many branches of science. Their mathematical behavior is also straightforward—adding scalars simply means summing their values.Yet, while scalars are everywhere, they are only part of the story. They tell us how much, but not in which direction. That's where vectors come in.A vector is fundamentally different from a scalar because it possesses both magnitude and direction. This added feature—direction—makes vectors more complex, but also far more informative.Take velocity, which is often confused with speed. Speed tells you how fast something is moving, but velocity tells you both how fast and in which direction. For instance, saying a plane is flying at 600 kilometers per hour is incomplete. But if you say it's flying at 600 km/h northeast, you have a full vector quantity: both magnitude and direction.Force is another vector. Push a book to the right across a table, and you apply a force with both a size (measured in newtons) and a direction. Acceleration, momentum, and displacement are also vector quantities.Vectors are visually represented as arrows. The length of the arrow shows the magnitude, and the arrowhead points in the direction. This visual tool becomes incredibly useful when multiple vectors interact—such as when combining wind direction with an airplane's heading, or analyzing forces acting on a bridge.In contrast to scalars, vectors require special mathematical handling. Adding vectors involves not just simple arithmetic but also geometry, trigonometry, and sometimes calculus. Yet this complexity allows vectors to model the real world with much greater fidelity.Imagine you're standing at a starting point and walk 5 kilometers north. This displacement is a vector—it tells both how far you went and in what direction. Now suppose you walk another 5 kilometers south. Your total distance traveled is 10 kilometers—a scalar. But your net displacement? Zero kilometers. You're right back where you started.This simple example illustrates why direction matters. Scalars like distance don't change based on direction. Vectors, however, are deeply tied to direction and therefore can cancel each other out even when magnitudes are high.This principle becomes crucial in understanding navigation, engineering, and any system involving multiple interacting forces. It's why a pilot needs to know not just wind speed (a scalar) but wind velocity (a vector). It's why structural engineers calculate not just how heavy something is, but how and where that weight is applied.To work with vectors effectively, we represent them mathematically, often using Cartesian coordinates. In a two-dimensional space, a vector might be written as v = (vx, vy), where vx is the component along the x-axis and vy is the component along the y-axis.Suppose you're moving northeast with a velocity of 10 m/s at a 45-degree angle. Using trigonometry, this can be broken down into:vx = 10 * cos(45°)vy = 10 * sin(45°)This results in approximately (7.07 m/s, 7.07 m/s). Now, instead of thinking of motion in a general direction, we have precise components that can be independently analyzed.Vector operations include addition, subtraction, scalar multiplication, and dot and cross products. Each operation reveals different aspects of how vectors interact.Adding vectors involves summing their components. If vector A = (3, 4) and vector B = (1, 2), then A + B = (4, 6). This corresponds to placing the tail of one vector at the head of another and drawing a new vector from the origin to the new head position.Subtraction works similarly, allowing us to determine relative motion or displacement. Multiplying a vector by a scalar scales its length but does not change its direction, much like turning up the speed of a car without changing where it's headed.Dot and cross products introduce deeper ideas. The dot product of two vectors gives a scalar and tells you how much one vector lies in the direction of another. The cross product gives another vector and is used extensively in physics to determine things like torque and magnetic force.Even though scalars don't have direction, they often play key roles in vector equations. For instance, when you multiply a vector by a scalar, you're scaling its magnitude without altering its direction.Consider Newton's second law of motion: F = ma. Here, force (F) is a vector, mass (m) is a scalar, and acceleration (a) is a vector. The scalar mass simply scales the acceleration vector, giving a force vector in the same direction as the acceleration.Energy equations also rely heavily on scalars. Kinetic energy is given by KE = ½mv², a scalar quantity derived from mass and speed (itself the scalar magnitude of velocity). Even when vectors are involved, the resulting quantity can be scalar, depending on the operation used.This interplay between scalars and vectors in physics is a delicate balance. Scalars may lack direction, but they are often the coefficients, constants, and parameters that shape vector behavior.The utility of vectors comes alive in applied physics. Consider planetary motion. Newton's law of universal gravitation describes a force between two masses, but it's the vector form that allows us to determine not just how strong that force is, but in which direction it pulls.In engineering, stress and strain are vector and tensor quantities. You can't just know how much force is acting on a structure—you must know the direction and point of application. Otherwise, buildings might buckle, bridges might fail, and airplanes might crash.In electromagnetism, vectors become essential. Electric and magnetic fields are vector fields; they have different strengths and directions at different points in space. Maxwell's equations, the cornerstone of electromagnetic theory, are vector equations that describe how these fields propagate, interact, and give rise to phenomena like light.Even in everyday life, vectors sneak in. The GPS in your phone calculates vector displacements to guide you. Weather maps show wind vectors to forecast storms. Video games use vectors for motion, collision detection, and physics simulations.In Einstein's theory of relativity, scalars and vectors evolve into more complex entities. Time, for instance, is a scalar in classical physics. But in special relativity, time becomes intertwined with space to form spacetime—a four-dimensional vector-like entity called a four-vector.Velocity in relativity is no longer simply distance over time but must be transformed according to relativistic rules. Scalars like mass also become frame-dependent; the concept of "relativistic mass" emerges, where mass increases with velocity.Quantum mechanics adds further layers. The wave function, central to quantum theory, is a complex scalar field—each point in space and time has an associated complex number. However, observables like momentum and angular momentum remain vector quantities, and their behavior is governed by operator algebra in vector spaces called Hilbert spaces.Even in this abstract mathematical landscape, the distinction between scalar and vector remains foundational. It affects how particles interact, how waves propagate, and how we understand the probabilistic nature of the universe.As physics advances, scalars and vectors prove insufficient for certain applications. Enter tensors, which are generalizations of vectors. A scalar is a rank-0 tensor, a vector is a rank-1 tensor, and more complex interactions—like stress in three dimensions—require rank-2 tensors.Tensor calculus is the language of general relativity. The curvature of spacetime itself is described using a tensor called the Riemann curvature tensor. This mathematical object encapsulates how mass and energy distort spacetime, producing what we perceive as gravity.Field theory also builds on vectors. A scalar field assigns a scalar value to every point in space (like temperature), while a vector field assigns a vector to every point (like wind velocity). These fields interact, evolve, and obey laws that are the bedrock of classical and modern physics.Understanding the scalar-vector distinction is often a student's first foray into mathematical physics. Yet, it can be an abstract hurdle. Educators increasingly use visual aids, interactive simulations, and real-world analogies to make the concepts intuitive.Seeing vectors as arrows, manipulating them on screen, and observing how they add or cancel helps develop a visceral understanding. Connecting these concepts to real phenomena—throwing a basketball, sailing a boat, riding a bike—bridges the gap between theory and lived experience.When students grasp not just the definition, but the why behind scalars and vectors, a deeper appreciation for physics follows. They begin to see the world not just as a collection of numbers, but as a dynamic system governed by elegant rules.Scalars and vectors may appear at first to be simple classifications, but they are the lens through which physics views the universe. They dictate how we model motion, energy, forces, and fields. Scalars give us the magnitude; vectors give us both magnitude and direction. Together, they form the vocabulary of physical law.Whether you are calculating how fast a car is moving or unraveling the equations that describe black holes, these two concepts will always be at your side. They are the first step toward understanding a cosmos that is complex, beautiful, and astonishingly structured.And so, when you next hear the terms "scalar" and "vector," don't dismiss them as dry jargon. They are the keys to unlocking motion, change, and reality itself—silent pillars of the physical world.Think this is important? Spread the knowledge! Share now. Sign Up Now &Daily Live Classes3000+ TestsStudy Material & PDFQuizzes With Detailed Analytics+ More BenefitsGet Free Access Now In everyday life, most people use the terms speed and velocity interchangeably, but to physicists, they're examples of two very different types of quantity. Mechanics problems deal with the motion of objects, and while you can just describe motion in terms of speed, the specific direction that something is going is often critically important. Similarly, the forces applied to objects can come from many different directions – think about the opposing pulls in a tug of war, for instance – so physicists describing situations like this need to use quantities that describe both the "size" of things like forces and the direction in which they act. These quantities are called vectors. A vector has both a magnitude and a specific direction, but a scalar quantity only has a magnitude. The key difference between vectors and scalars is that a vector's magnitude doesn't entirely describe it; there also needs to be a stated direction. The direction of a vector can be stated in numerous ways, whether through positive or negative signs in front of it, expressing it in the form of components (scalar values next to the appropriate i, **j** and k "unit vector," which correspond to the Cartesian coordinates of x, y and z, respectively), adding an angle with respect to a stated direction (e.g., "60 degrees from the x-axis") or simply adding some words to describe the direction (e.g., "northwest"). By contrast, a scalar is just the vector's magnitude without any additional notation or information provided – for example, speed is a scalar equivalent of the velocity vector. From a mathematical perspective, it's the absolute value of the vector. However, many quantities, such as energy, pressure, length, mass, power and temperature are examples of scalars that aren't just the magnitude of a corresponding vector. You don't need to know the "direction" of mass, for example, to have a complete picture of it as a physical property. There are a few counterintuitive facts that you can understand when you know the difference between a scalar and a vector, such as the idea that something could have a constant speed but a continuously changing velocity. Imagine a car driving at a constant speed of 10 km/h but in a circle. Because the direction of a vector is part of its definition, the car's velocity vector is always changing in this example, despite the fact that the magnitude of the vector (i.e., its speed) is constant. There are many examples of vectors in physics, but some of the most well-known examples are force, momentum, acceleration and velocity, all of which feature strongly in classical physics. A velocity vector could be displayed as 25 m/s to the east, −8 km/h in the y-direction, **v** = 5 m/s i + 10 m/s j, or 10 m/s in a direction 50 degrees from the x-axis. Momentum vectors are another example you can use to see how the magnitude and direction of the vector are displayed in physics. These work just like the velocity vector examples, with 50 kg m/s to the west, −12 km/h in the z direction, **p** = 12 kg m/s i – 10 kg m/s j – 15 kg m/s k and 100 kg m/s 30 degrees from the x-axis being examples of how they could be displayed. The same basic points go for the display of acceleration vectors, with the only difference being the unit of m/s2 and the commonly-used symbol for the vector, a. Force is the final one of these examples of vector expressions, and while there are many similarities, using cylindrical coordinates (r, θ, z) instead of Cartesian coordinates can help to show other ways they may be displayed. For example, you might write a force as **F** = 10 N r + 35 N ****, for a force with components in the radial direction and the azimuthal direction, or describe the force of gravity on a 1-kg object on Earth as 10 N in the –r direction (i.e., towards the center of the planet). In diagrams, vectors are displayed using arrows, with the magnitude of the vector represented by the length of the arrow and its direction represented by the direction in which the arrow points. For example, a larger arrow shows that a force is larger (i.e., more newtons or a bigger magnitude) than another force. For a vector that shows motion, such as the momentum or the velocity vector, the zero vector (i.e., a vector representing no velocity or momentum) is displayed using a single dot. It's worth noting that because the length of the arrow represents the magnitude of the vector and its orientation represents the direction of the vector. It's useful to try to be reasonably accurate when making a vector diagram. It doesn't have to be perfect, but if the vector a is twice as big as the vector b, the arrow should be roughly twice as long. Vector addition and vector subtraction are a bit more complicated than adding and subtracting scalars, but you can pick up the concepts easily. There are two main approaches you can use, and each has potential uses depending on the specific problem you're tackling. The first, and the easiest to use when you've been given two vectors in component form, is to simply add matching components in the same way you'd add ordinary scalars. For example, if you needed to add the two forces **F**1 = 5 N i + 10 N j and **F**2 = 6 N i + 15 N j + 10 N k, you would add the i components, then the j components and finally the k components as follows: \(\begin{aligned} \bm{F}_1 + \bm{F}_2 &= (5 \;\text{N} \;\bold{i} + 10 \;\text{N}\;\bold{j}) + (6 \;\text{N} \;\bold{i} + 15 \;\text{N}\;\bold{j} + 10 \;\text{N}\;\bold{k})\\ \(&= (5 \;\text{N} + 6 \;\text{N}) \bold{i} + (10 \;\text{N} + 15 \;\text{N}) \bold{j} + (0 \;\text{N} + 10 \;\text{N}) \bold{k}\\ \(&= 11 \;\text{N} \;\bold{i} + 25 \;\text{N} \;\bold{j} + 10 \;\text{N} \;\bold{k} \end{aligned}\) Vector subtraction works in exactly the same way, except you subtract the quantities rather than add them. Vector addition is also commutative, like ordinary addition with real numbers, so **a **+ **b** = **b** + **a**. You can also perform vector addition using arrow diagrams by laying the vector arrows head to tail and then drawing a new vector arrow for the sum of the vectors connecting the tail of the first arrow with the head of the second. If you have a simple vector addition with one in the x-direction and another in the y-direction, the diagram forms a right-angled triangle. You can complete the vector addition and determine the resulting vector's magnitude and direction by "solving" the triangle using trigonometry and Pythagoras' theorem. Multiplying vectors is a bit more complicated than scalar multiplication for real numbers, but the two main forms of multiplication are the dot product and the cross product. The dot product is called the scalar product and is defined as: \(\bm{u} \;• \;\bm{v} = u_1v_1 + u_2v_2 + u_3v_3\) or \(\bm{u} \;• \;\bm{v} = \lvert\bm{u}\rvert\lvert\bm{v}\rvert \text{cos}(θ)\) where θ is the angle between the two vectors, and the subscripts 1, 2 and 3 represent the first, second and third component of the vector. The result of the dot product is a scalar. The cross product is defined as: \(\bm{a} \; \bold{×} \;\bm{b} =(a_2b_3 − a_3b_2, a_3b_1 − a_1b_3,a_1b_2 − a_2b_1)\) with the commas separating components of the result in different directions. Johnson, Lee. "Vectors & Scalars: What Are They & Why They Matter?" sciencing.com, . 28 December 2020. APA Johnson, Lee. (2020, December 28). Vectors & Scalars: What Are They & Why They Matter?. sciencing.com. Retrieved from Chicago Johnson, Lee. Vectors & Scalars: What Are They & Why They Matter? last modified August 30, 2022. By the end of this section, you will be able to: Describe the difference between vector and scalar quantities. Identify the magnitude and direction of a vector. Explain the effect of multiplying a vector quantity by a scalar. Describe how one-dimensional vector quantities are added or subtracted. Explain the geometric construction for the addition or subtraction of vectors in a plane. Distinguish between a vector equation and a scalar equation. Many familiar physical quantities can be specified completely by giving a single number and the appropriate unit. For example, "a class period lasts 50 min" or "the gas tank in my car holds 65 L" or "the distance between two posts is 100 m." A physical quantity that can be specified completely in this manner is called a scalar quantity. Scalar is a synonym of "number." Time, mass, distance, length, volume, temperature, and energy are examples of scalar quantities. Scalar quantities that have the same physical units can be added or subtracted according to the usual rules of algebra for numbers. For example, a class ending 10 min earlier than 50 min lasts $$ 50\,\text{min}-10\,\text{min}=40\,\text{min}$$. Similarly, a 60-cal serving of corn followed by a 200-cal serving of donuts gives $$ 60\,\text{cal}+200\,\text{cal}=260\,\text{cal} $$ of energy. When we multiply a scalar quantity by a number, we obtain the same scalar quantity but with a larger (or smaller) value. For example, if yesterday's breakfast had 200 cal of energy and today's breakfast has four times as much energy as it had yesterday, then today's breakfast has $$ 4(200\,\text{cal})=800\,\text{cal} $$ of energy. Two scalar quantities can also be multiplied or divided by each other to form a derived scalar quantity. For example, if a train covers a distance of 100 km in 1.0 h, its speed is 100.0 km/1.0 h = 27.8 m/s, where the speed is a derived scalar quantity obtained by dividing distance by time. Many physical quantities, however, cannot be described completely by just a single number of physical units. For example, when the U.S. Coast Guard dispatches a ship or a helicopter for a rescue mission, the rescue team must know not only the distance to the distress signal, but also the direction from which the signal is coming so they can get to its origin as quickly as possible. Physical quantities specified completely by giving a number of units (magnitude) and a direction are called vector quantities. Examples of vector quantities include displacement, velocity, position, force, and torque. In the language of mathematics, physical vector quantities are represented by mathematical objects called vectors ((Figure)). We can add or subtract two vectors, and we can multiply a vector by a scalar or by another vector, but we cannot divide by a vector. The operation of division by a vector is not defined. Figure 2.2 We draw a vector from the initial point or origin (called the "tail" of a vector) to the end or terminal point (called the "head" of a vector), marked by an arrowhead. Magnitude is the length of a vector and is always a positive scalar quantity. (credit: modification of work by Cate Sevilla) Let's examine vector algebra using a graphical method to be aware of basic terms and to develop a qualitative understanding. In practice, however, when it comes to solving physics problems, we use analytical methods, which we'll see in the next section. Analytical methods are more simple computationally and more accurate than graphical methods. From now on, to distinguish between a vector and a scalar quantity, we adopt the common convention that a letter in bold type with an arrow above it denotes a vector, and a letter without an arrow denotes a scalar. For example, a distance of 2.0 km, which is a scalar quantity, is denoted by d = 2.0 km, whereas a displacement of 2.0 km in some direction, which is a vector quantity, is denoted by $$ \overset{\to }{d}$$. Suppose you tell a friend on a camping trip that you have discovered a terrific fishing hole 6 km from your tent. It is unlikely your friend would be able to find the hole easily unless you also communicate the direction in which it can be found with respect to your campsite. You may say, for example, "Walk about 6 km northeast from my tent." The key concept here is that you have to give not one but two pieces of information—namely, the distance or magnitude (6 km) and the direction (northeast). Displacement is a general term used to describe a change in position, such as during a trip from the tent to the fishing hole. Displacement is an example of a vector quantity. If you walk from the tent (location A) to the hole (location B), as shown in (Figure), the vector $$ \overset{\to }{D}$$, representing your displacement, is drawn as the arrow that originates at point A and ends at point B. The arrowhead marks the end of the vector. The direction of the displacement vector $$ \overset{\to }{D} $$ is the direction of the arrow. The length of the arrow represents the magnitude D of vector $$ \overset{\to }{D}$$. Here, D = 6 km. Since the magnitude of a vector is its length, which is a positive number, the magnitude is also indicated by placing the absolute value notation around the symbol that denotes the vector; so, we can write equivalently that $$ D\equiv |\overset{\to }{D}|$$. To solve a vector problem graphically, we need to draw the vector $$ \overset{\to }{D} $$ to scale. For example, if we assume 1 unit of distance (1 km) is represented in the drawing by a line segment of length u = 2 cm, then the total displacement in this example is represented by a vector of length $$ d=6u=6(2\,\text{cm})=12\,\text{cm}$$, as shown in (Figure). Notice that here, to avoid confusion, we used $$ D=6\,\text{km} $$ to denote the magnitude of the actual displacement and d = 12 cm to denote the length of its representation in the drawing. Figure 2.3 The displacement vector from point A (the initial position at the campsite) to point B (the final position at the fishing hole) is indicated by an arrow with origin at point A and end at point B. The displacement is the same for any of the actual paths (dashed curves) that may be taken between points A and B. Figure 2.4 A displacement $$ \overset{\to }{D} $$ of magnitude 6 km is drawn to scale as a vector of length 12 cm when the length of 2 cm represents 1 unit of displacement (which in this case is 1 km). Suppose your friend walks from the campsite at A to the fishing pond at B and then walks back: from the fishing pond at B to the campsite at A. The magnitude of the displacement vector $$ {\overset{\to }{D}}_{AB} $$ from A to B is the same as the magnitude of the displacement vector $$ {\overset{\to }{D}}_{BA} $$ from B to A (it equals 6 km in both cases), so we can write $$ {D}_{AB}={D}_{BA}$$. However, vector $$ {\overset{\to }{D}}_{AB} $$ is not equal to vector $$ {\overset{\to }{D}}_{BA} $$ because these two vectors have different directions: $$ {\overset{\to }{D}}_{AB}e {\overset{\to }{D}}_{BA}$$. In (Figure), vector $$ {\overset{\to }{D}}_{BA} $$ would be represented by a vector with an origin at point B and an end at point A, indicating vector $$ {\overset{\to }{D}}_{BA} $$ points to the southwest, which is exactly $$ 180\text{°} $$ opposite to the direction of vector $$ {\overset{\to }{D}}_{AB}$$. We say that vector $$ {\overset{\to }{D}}_{BA} $$ is antiparallel to vector $$ {\overset{\to }{D}}_{AB} $$ and write $$ {\overset{\to }{D}}_{AB}=\text{−}{\overset{\to }{D}}_{BA}$$, where the minus sign indicates the antiparallel direction. Two vectors that have identical directions are said to be parallel vectors—meaning, they are parallel to each other. Two parallel vectors $$ \overset{\to }{A} $$ and $$ \overset{\to }{B} $$ are equal, denoted by $$ \overset{\to }{A}=\overset{\to }{B}$$, if and only if they have equal magnitudes $$ |\overset{\to }{A}|=|\overset{\to }{B}|$$. Two vectors with directions perpendicular to each other are said to be orthogonal vectors. These relations between vectors are illustrated in (Figure). Figure 2.5 Various relations between two vectors $$ \overset{\to }{A} $$ and $$ \overset{\to }{B}$$. (a) $$ \overset{\to }{A}e

\overset{\to }{B} $$ because $$ Ae B$$. (b) $$ \overset{\to }{A}e \overset{\to }{B} $$ because they are not parallel and $$ Ae B$$. (c) $$ \overset{\to }{A}e \text{−}\overset{\to }{A} $$ because they have different directions (even though $$ |\overset{\to }{A}|=|-\overset{\to }{A}|=A)$$. (d) $$ \overset{\to }{A}=\overset{\to }{B} $$ because they are parallel and have identical magnitudes A = B. (e) $$ \overset{\to }{A}e \overset{\to }{B} $$ because they have different directions (are not parallel); here, their directions differ by $$ 90\text{°}$$—meaning, they are orthogonal. Two motorboats named Alice and Bob are moving on a lake. Given the information about their velocity vectors in each of the following situations, indicate whether their velocity vectors are equal or otherwise. (a) Alice moves north at 6 knots and Bob moves west at 6 knots. (b) Alice moves west at 6 knots and Bob moves south at 3 knots. (c) Alice moves northeast at 6 knots and Bob moves south at 3 knots. (d) Alice moves northeast at 6 knots and Bob moves southwest at 6 knots. (e) Alice moves northeast at 2 knots and Bob moves closer to the shore northeast at 2 knots. Vectors can be multiplied by scalars, added to other vectors, or subtracted from other vectors. We can illustrate these vector concepts using an example of the fishing trip seen in (Figure). Figure 2.6 Displacement vectors for a fishing trip. (a) Stopping to rest at point C while walking from camp (point A) to the pond (point B). (b) Going back for the dropped tackle box (point D). (c) Finishing up at the fishing pond. Suppose your friend departs from point A (the campsite) and walks in the direction to point B (the fishing pond), but, along the way, stops to rest at some point C located three-quarters of the distance between A and B, beginning from point A ((Figure)(a)). What is his displacement vector $$ {\overset{\to }{D}}_{AC} $$ when he reaches point C? We know that if he walks all the way to B, his displacement vector relative to A is $$ {\overset{\to }{D}}_{AB}$$, which has magnitude $$ {D}_{AB}=6\,\text{km} $$ and a direction of northeast. If he walks only a 0.75 fraction of the total distance, maintaining the northeasterly direction, at point C he must be $$ 0.75{D}_{AB}=4.5\,\text{km} $$ away from the campsite at A. So, his displacement vector at the rest point C has magnitude $$ {D}_{AC}=4.5\,\text{km}=0.75{D}_{AB} $$ and is parallel to the displacement vector $$ {\overset{\to }{D}}_{AB}$$. All of this can be stated succinctly in the form of the following vector equation: $${\overset{\to }{D}}_{AC}=0.75{\overset{\to }{D}}_{AB}.$$ In a vector equation, both sides of the equation are vectors. The previous equation is an example of a vector multiplied by a positive scalar (number) $$ \alpha =0.75$$. The result, $$ {\overset{\to }{D}}_{AC}$$, of such a multiplication is a new vector with a direction parallel to the direction of the original vector $$ {\overset{\to }{D}}_{AB}$$. In general, when a vector $$ \overset{\to }{A} $$ is multiplied by a positive scalar $$ \alpha $$, the result is a new vector $$ \overset{\to }{B} $$ that is parallel to $$ \overset{\to }{A}$$: $$\overset{\to }{B}=\alpha \overset{\to }{A}.$$ The magnitude $$ |\overset{\to }{B}| $$ of this new vector is obtained by multiplying the magnitude $$ |\overset{\to }{A}| $$ of the original vector, as expressed by the scalar equation: In a scalar equation, both sides of the equation are numbers. (Figure) is a scalar equation because the magnitudes of vectors are scalar quantities (and positive numbers). If the scalar $$ \alpha $$ is negative in the vector equation (Figure), then the magnitude $$ |\overset{\to }{B}| $$ of the new vector is still given by (Figure), but the direction of the new vector $$ \overset{\to }{B} $$ is antiparallel to the direction of $$ \overset{\to }{A}$$. These principles are illustrated in (Figure)(a) by two examples where the length of vector $$ \overset{\to }{A} $$ is 1.5 units. When $$ \alpha =2$$, the new vector $$ \overset{\to }{B}=2\overset{\to }{A} $$ has length $$ B=2A=3.0\,\text{units} $$ (twice as long as the original vector) and is parallel to the original vector. When $$ \alpha =-2$$, the new vector $$ \overset{\to }{C}=-2\overset{\to }{A} $$ has length $$ C=|-2|A=3.0\,\text{units} $$ (twice as long as the original vector) and is antiparallel to the original vector. Figure 2.7 Algebra of vectors in one dimension. (a) Multiplication by a scalar. (b) Addition of two vectors $$ (\overset{\to }{R} $$ is called the resultant of vectors $$ \overset{\to }{A} $$ and $$ \overset{\to }{B})$$. (c) Subtraction of two vectors $$ (\overset{\to }{D} $$ is the difference of vectors $$ \overset{\to }{A} $$ and $$ \overset{\to }{B})$$. Now suppose your fishing buddy departs from point A (the campsite), walking in the direction to point B (the fishing hole), but he realizes he lost his tackle box when he stopped to rest at point C (located three-quarters of the distance between A and B, beginning from point A). So, he turns back and retraces his steps in the direction toward the campsite and finds the box lying on the path at some point D only 1.2 km away from point C (see (Figure)(b)). What is his displacement vector $$ {\overset{\to }{D}}_{AD} $$ when he finds the box at point D? What is his displacement vector $$ {\overset{\to }{D}}_{DB} $$ from point D to the hole? We have already established that at rest point C his displacement vector is $$ {\overset{\to }{D}}_{AC}=0.75{\overset{\to }{D}}_{AB}.$$ Starting at point C, he walks southwest (toward the campsite), which means his new displacement vector $$ {\overset{\to }{D}}_{CD} $$ from point C to point D is antiparallel to $$ {\overset{\to }{D}}_{AB}$$. Its magnitude $$ |{\overset{\to }{D}}_{CD}| $$ is $$ {D}_{CD}=1.2\,\text{km}=0.2{D}_{AB}$$, so his second displacement vector is $$ {\overset{\to }{D}}_{CD}=-0.2{\overset{\to }{D}}_{AB}$$. His total displacement $$ {\overset{\to }{D}}_{AD} $$ relative to the campsite is the vector sum of the two displacement vectors: vector $$ {\overset{\to }{D}}_{AC} $$ (from the campsite to the rest point) and vector $$ {\overset{\to }{D}}_{CD} $$ (from the rest point to the point where he finds his box): $${\overset{\to }{D}}_{AD}={\overset{\to }{D}}_{AC}+{\overset{\to }{D}}_{CD}.$$ The vector sum of two (or more) vectors is called the resultant vector or, for short, the resultant. When the vectors on the right-hand-side of (Figure) are known, we can find the resultant $$ {\overset{\to }{D}}_{AD} $$ as follows: $${\overset{\to }{D}}_{AD}={\overset{\to }{D}}_{AC}+{\overset{\to }{D}}_{CD}=0.75{\overset{\to }{D}}_{AB}-0.2{\overset{\to }{D}}_{AB}=(0.75-0.2){\overset{\to }{D}}_{AB}=0.55{\overset{\to }{D}}_{AB}.$$ When your friend finally reaches the pond at B, his displacement vector $$ {\overset{\to }{D}}_{AB} $$ from point A is the vector sum of his displacement vector $$ {\overset{\to }{D}}_{AD} $$ from point A to point D and his displacement vector $$ {\overset{\to }{D}}_{DB} $$ from point D to the fishing hole: $$ {\overset{\to }{D}}_{AB}={\overset{\to }{D}}_{AD}+{\overset{\to }{D}}_{DB} $$ (see (Figure)(c)). This means his displacement vector $$ {\overset{\to }{D}}_{DB} $$ is the difference of two vectors: $${\overset{\to }{D}}_{DB}={\overset{\to }{D}}_{AB}-{\overset{\to }{D}}_{AD}={\overset{\to }{D}}_{AB}+(\text{−}{\overset{\to }{D}}_{AD}).$$ Notice that a difference of two vectors is nothing more than a vector sum of two vectors because the second term in (Figure) is vector $$ \text{−}{\overset{\to }{D}}_{AD} $$ (which is antiparallel to $$ {\overset{\to }{D}}_{AD})$$. When we substitute (Figure) into (Figure), we obtain the second displacement vector: $${\overset{\to }{D}}_{DB}={\overset{\to }{D}}_{AB}-{\overset{\to }{D}}_{AD}={\overset{\to }{D}}_{AB}-0.55{\overset{\to }{D}}_{AB}=(1.0-0.55){\overset{\to }{D}}_{AB}=0.45{\overset{\to }{D}}_{AB}.$$ This result means your friend walked $$ {D}_{DB}=0.45{D}_{AB}=0.45(6.0\,\text{km})=2.7\,\text{km} $$ from the point where he finds his tackle box to the fishing hole. When vectors $$ \overset{\to }{A} $$ and $$ \overset{\to }{B} $$ lie along a line (that is, in one dimension), such as in the camping example, their resultant $$ \overset{\to }{R}=\overset{\to }{A}+\overset{\to }{B} $$ and their difference $$ \overset{\to }{D}=\overset{\to }{A}-\overset{\to }{B} $$ both lie along the same direction. We can illustrate the addition or subtraction of vectors by drawing the corresponding vectors to scale in one dimension, as shown in (Figure). To illustrate the resultant when $$ \overset{\to }{A} $$ and $$ \overset{\to }{B} $$ are two parallel vectors, we draw them along one line by placing the origin of one vector at the end of the other vector in head-to-tail fashion (see (Figure)(b)). The magnitude of this resultant is the sum of their magnitudes: R = A + B. The direction of the resultant is parallel to both vectors. When vector $$ \overset{\to }{A} $$ is antiparallel to vector $$ \overset{\to }{B}$$, we draw them along one line in either head-to-head fashion ((Figure)(c)) or tail-to-tail fashion. The magnitude of the vector difference, then, is the absolute value $$ D=|A-B| $$ of the difference of their magnitudes. The direction of the difference vector $$ \overset{\to }{D} $$ is parallel to the direction of the longer vector. In general, in one dimension—as well as in higher dimensions, such as in a plane or in space—we can add any number of vectors and we can do so in any order because the addition of vectors is commutative, $$\overset{\to }{A}+\overset{\to }{B}=\overset{\to }{B}+\overset{\to }{A},$$ and associative, $$(\overset{\to }{A}+\overset{\to }{B})+\overset{\to }{C}=\overset{\to }{A}+(\overset{\to }{B}+\overset{\to }{C}).$$ Moreover, multiplication by a scalar is distributive: $${\alpha }_{1}\overset{\to }{A}+{\alpha }_{2}\overset{\to }{A}=({\alpha }_{1}+{\alpha }_{2})\overset{\to }{A}.$$ We used the distributive property in (Figure) and (Figure). When adding many vectors in one dimension, it is convenient to use the concept of a unit vector. A unit vector, which is denoted by a letter symbol with a hat, such as $$ \hat{u}$$, has a magnitude of one and does not have any physical unit so that $$ |\hat{u}|\equiv u=1$$. The only role of a unit vector is to specify direction. For example, instead of saying vector $$ {\overset{\to }{D}}_{AB} $$ has a magnitude of 6.0 km and a direction of northeast, we can introduce a unit vector $$ \hat{u} $$ that points to the northeast and say succinctly that $$ {\overset{\to }{D}}_{AB}=(6.0\,\text{km})\hat{u}$$. Then the southwesterly direction is simply given by the unit vector $$ \text{−}\hat{u}$$. In this way, the displacement of 6.0 km in the southwesterly direction is expressed by the vector $${\overset{\to }{D}}_{BA}=(-6.0\,\text{km})\hat{u}.$$ A long measuring stick rests against a wall in a physics laboratory with its 200-cm end at the floor. A ladybug lands on the 100-cm mark and crawls randomly along the stick. It first walks 15 cm toward the floor, then it walks 56 cm toward the wall, then it walks 3 cm toward the floor again. Then, after a brief stop, it continues for 25 cm toward the floor and then, again, it crawls up 19 cm toward the wall before coming to a complete rest ((Figure)). Find the vector of its total displacement and its final resting position on the stick. Strategy If we choose the direction along the stick toward the floor as the direction of unit vector $$ \hat{u}$$, then the direction toward the floor is $$ +\hat{u} $$ and the direction toward the wall is $$ \text{−}\hat{u}$$. The ladybug makes a total of five displacements: $$\begin{array}{c}{\overset{\to }{D}}_{1}=(15\,\text{cm})(+\hat{u}),\hfill \\ {\overset{\to }{D}}_{2}=(56\,\text{cm})(\text{−}\hat{u}),\hfill \\ {\overset{\to }{D}}_{3}=(3\,\text{cm})(+\hat{u}),\hfill \\ {\overset{\to }{D}}_{4}=(25\,\text{cm})(+\hat{u}),\,\text{and}\hfill \\ {\overset{\to }{D}}_{5}=(19\,\text{cm})(\text{−}\hat{u}).\hfill \end{array}$$ The total displacement $$ \overset{\to }{D} $$ is the resultant of all its displacement vectors. Figure 2.8 Five displacements of the ladybug. Note that in this schematic drawing, magnitudes of displacements are not drawn to scale. (credit: modification of work by "Persian Poet Gal"/Wikimedia Commons) Solution Show Answer A cave diver enters a long underwater tunnel. When her displacement with respect to the entry point is 20 m, she accidentally drops her camera, but she doesn't notice it missing until she is some 6 m farther into the tunnel. She swims back 10 m but cannot find the camera, so she decides to end the dive. How far from the entry point is she? Taking the positive direction out of the tunnel, what is her displacement vector relative to the entry point? When vectors lie in a plane—that is, when they are in two dimensions—they can be multiplied by scalars, added to other vectors, or subtracted from other vectors in accordance with the general laws expressed by (Figure), (Figure), (Figure), and (Figure). However, the addition rule for two vectors in a plane becomes more complicated than the rule for vector addition in one dimension. We have to use the laws of geometry to construct resultant vectors, followed by trigonometry to find vector magnitudes and directions. This geometric approach is commonly used in navigation ((Figure)). In this section, we need to have at hand two rulers, a triangle, a protractor, a pencil, and an eraser for drawing vectors to scale by geometric constructions. For a geometric construction of the sum of two vectors in a plane, we follow the parallelogram rule. Suppose two vectors $$ \overset{\to }{A} $$ and $$ \overset{\to }{B} $$ are at the arbitrary positions shown in (Figure). Translate either one of them in parallel to the beginning of the other vector, so that after the translation, both vectors have their origins at the same point. Now, at the end of vector $$ \overset{\to }{A} $$ we draw a line parallel to vector $$ \overset{\to }{B} $$ and at the end of vector $$ \overset{\to }{B} $$ we draw a line parallel to vector $$ \overset{\to }{A} $$ (the dashed lines in (Figure)). In this way, we obtain a parallelogram. From the origin of the two vectors we draw a diagonal that is the resultant $$ \overset{\to }{R} $$ of the two vectors: $$ \overset{\to }{R}=\overset{\to }{A}+\overset{\to }{B} $$ ((Figure)(a)). The other diagonal of this parallelogram is the vector difference of the two vectors $$ \overset{\to }{D}=\overset{\to }{A}-\overset{\to }{B}$$, as shown in (Figure)(b). Notice that the end of the difference vector is placed at the end of vector $$ \overset{\to }{A}$$. Figure 2.10 The parallelogram rule for the addition of two vectors. Make the parallel translation of each vector to a point where their origins (marked by the dot) coincide and construct a parallelogram with two sides on the vectors and the other two sides (indicated by dashed lines) parallel to the vectors. (a) Draw the resultant vector $$ \overset{\to }{R} $$ along the diagonal of the parallelogram from the common point to the opposite corner. Length R of the resultant vector is not equal to the sum of the magnitudes of the two vectors. (b) Draw the difference vector $$ \overset{\to }{D}=\overset{\to }{A}-\overset{\to }{B} $$ along the diagonal connecting the ends of the vectors. Place the origin of vector $$ \overset{\to }{D} $$ at the end of vector $$ \overset{\to }{B} $$ and the end (arrowhead) of vector $$ \overset{\to }{D} $$ at the end of vector $$ \overset{\to }{A}$$. Length D of the difference vector is not equal to the difference of magnitudes of the two vectors. It follows from the parallelogram rule that neither the magnitude of the resultant vector nor the magnitude of the difference vector can be expressed as a simple sum or difference of magnitudes A and B, because the length of a diagonal cannot be expressed as a simple sum of side lengths. When using a geometric construction to find magnitudes $$ |\overset{\to }{R}| $$ and $$ |\overset{\to }{D}|$$, we have to use trigonometry laws for triangles, which may lead to complicated algebra. There are two ways to circumvent this algebraic complexity. One way is to use the method of components, which we examine in the next section. The other way is to draw the vectors to scale, as is done in navigation, and read approximate vector lengths and angles (directions) from the graphs. In this section we examine the second approach. If we need to add three or more vectors, we repeat the parallelogram rule for the pairs of vectors until we find the resultant of all of the resultants. For three vectors, for example, we first find the resultant of vector 1 and vector 2, and then we find the resultant of this resultant and vector 3. The order in which we select the pairs of vectors does not matter because the operation of vector addition is commutative and associative (see (Figure) and (Figure)). Before we state a general rule that follows from repetitive applications of the parallelogram rule, let's look at the following example. Suppose you plan a vacation trip in Florida. Departing from Tallahassee, the state capital, you plan to visit your uncle Joe in Jacksonville, see your cousin Vinny in Daytona Beach, stop for a little fun in Orlando, see a circus performance in Tampa, and visit the University of Florida in Gainesville. Your route may be represented by five displacement vectors $$ \overset{\to }{A}, $$ $$\overset{\to }{B}$$, $$ \overset{\to }{C}$$, $$ \overset{\to }{D}$$, and $$ \overset{\to }{E}$$, which are indicated by the red vectors in (Figure). What is your total displacement when you reach Gainesville? The total displacement is the vector sum of all five displacement vectors, which may be found by using the parallelogram rule four times. Alternatively, recall that the displacement vector has its beginning at the initial position (Tallahassee) and its end at the final position (Gainesville), so the total displacement vector can be drawn directly as an arrow connecting Tallahassee with Gainesville (see the green vector in (Figure)). When we use the parallelogram rule four times, the resultant $$ \overset{\to }{R} $$ we obtain is exactly this green vector connecting Tallahassee with Gainesville: $$ \overset{\to }{R}=\overset{\to }{A}+\overset{\to }{B}+\overset{\to }{C}+\overset{\to }{D}+\overset{\to }{E}$$. Figure 2.11 When we use the parallelogram rule four times, we obtain the resultant vector $$ \overset{\to }{R}=\overset{\to }{A}+\overset{\to }{B}+\overset{\to }{C}+\overset{\to }{D}+\overset{\to }{E}$$, which is the green vector connecting Tallahassee with Gainesville. Drawing the resultant vector of many vectors can be generalized by using the following tail-to-head geometric construction. Suppose we want to draw the resultant vector $$ \overset{\to }{R} $$ of four vectors $$ \overset{\to }{A}$$, $$ \overset{\to }{B}$$, $$ \overset{\to }{C}$$, and $$ \overset{\to }{D} $$ ((Figure)(a)). We select any one of the vectors as the first vector and make a parallel translation of a second vector to a position where the origin ("tail") of the second vector coincides with the end ("head") of the first vector. Then, we select a third vector and make a parallel translation of the third vector to a position where the origin of the third vector coincides with the end of the second vector. We repeat this procedure until all the vectors are in a head-to-tail arrangement like the one shown in (Figure). We draw the resultant vector $$ \overset{\to }{R} $$ by connecting the origin ("tail") of the first vector with the end ("head") of the last vector. The end of the resultant vector is at the end of the last vector. Because the addition of vectors is associative and commutative, we obtain the same resultant vector regardless of which vector we choose to be first, second, third, or fourth in this construction. Figure 2.12 Tail-to-head method for drawing the resultant vector $$ \overset{\to }{R}=\overset{\to }{A}+\overset{\to }{B}+\overset{\to }{C}+\overset{\to }{D}$$. (a) Four vectors of different magnitudes and directions. (b) Vectors in (a) are translated to new positions where the origin ("tail") of one vector is at the end ("head") of another vector. The resultant vector is drawn from the origin ("tail") of the first vector to the end ("head") of the last vector in this arrangement. The three displacement vectors $$ \overset{\to }{A}$$, $$ \overset{\to }{B}$$, and $$ \overset{\to }{C} $$ in (Figure) are specified by their magnitudes A = 10.0, B = 7.0, and C = 8.0, respectively, and by their respective direction angles with the horizontal direction $$ \alpha =35\text{°}$$, $$ \beta =-110\text{°}$$, and $$ \gamma =30\text{°}$$. The physical units of the magnitudes are centimeters. Choose a convenient scale and use a ruler and a protractor to find the following vector sums: (a) $$ \overset{\to }{R}=\overset{\to }{A}+\overset{\to }{B}$$, (b) $$ \overset{\to }{D}=\overset{\to }{A}-\overset{\to }{B}\text{, and} $$ (c) $$ \overset{\to }{S}=\overset{\to }{A}-3\overset{\to }{B}+\overset{\to }{C}$$. Figure 2.13 Vectors used in (Figure) and in the Check Your Understanding feature that follows. Strategy In geometric construction, to find a vector means to find its magnitude and its direction angle with the horizontal direction. The strategy is to draw to scale the vectors that appear on the right-hand side of the equation and construct the resultant vector. Then, use a ruler and a protractor to read the magnitude of the resultant and the direction angle. For parts (a) and (b) we use the parallelogram rule. For (c) we use the tail-to-head method. Solution For parts (a) and (b), we attach the origin of vector $$ \overset{\to }{B} $$ to the origin of vector $$ \overset{\to }{A}$$, as shown in (Figure), and construct a parallelogram. The shorter diagonal of this parallelogram is the sum $$ \overset{\to }{A}+\overset{\to }{B}$$. The longer of the diagonals is the difference $$ \overset{\to }{A}-\overset{\to }{B}$$. We use a ruler to measure the lengths of the diagonals, and a protractor to measure the angles with the horizontal. For the resultant $$ \overset{\to }{R}$$, we obtain R = 5.8 cm and $$ {\theta }_{R}\approx 0\text{°}$$. For the difference $$ \overset{\to }{D}$$, we obtain D = 16.2 cm and $$ {\theta }_{D}=49.3\text{°}$$, which are shown in (Figure). Figure 2.14 Using the parallelogram rule to solve (a) (finding the resultant, red) and (b) (finding the difference, blue). For (c), we can start with vector $$ -3\overset{\to }{B} $$ and draw the remaining vectors tail-to-head as shown in (Figure). In vector addition, the order in which we draw the vectors is unimportant, but drawing the vectors to scale is very important. Next, we draw vector $$ \overset{\to }{S} $$ from the origin of the first vector to the end of the last vector and place the arrowhead at the end of $$ \overset{\to }{S}$$. We use a ruler to measure the length of $$ \overset{\to }{S}$$, and find that its magnitude is Figure 2.15 Using the tail-to-head method to solve (c) (finding vector $$ \overset{\to }{S}$$, green). Using the three displacement vectors $$ \overset{\to }{A}$$, $$ \overset{\to }{B}$$, and $$ \overset{\to }{F} $$ in (Figure), choose a convenient scale, and use a ruler and a protractor to find vector $$ \overset{\to }{G} $$ given by the vector equation $$ \overset{\to }{G}=\overset{\to }{A}+2\overset{\to }{B}-\overset{\to }{F}$$. A weather forecast states the temperature is predicted to be $$ -5\,\text{°}\text{C} $$ the following day. Is this temperature a vector or a scalar quantity? Explain. Which of the following is a vector: a person's height, the altitude on Mt. Everest, the velocity of a fly, the age of Earth, the boiling point of water, the cost of a book, Earth's population, or the acceleration of gravity? Give a specific example of a vector, stating its magnitude, units, and direction. What do vectors and scalars have in common? How do they differ? Suppose you add two vectors $$ \overset{\to }{A} $$ and $$ \overset{\to }{B}$$. What relative direction between them produces the resultant with the greatest magnitude? What is the maximum magnitude? What relative direction between them produces the resultant with the smallest magnitude? What is the minimum magnitude? Is it possible to add a scalar quantity to a vector quantity? Is it possible for two vectors of different magnitudes to add to zero? Is it possible for three vectors of different magnitudes to add to zero? Explain. Does the odometer in an automobile indicate a scalar or a vector quantity? When a 10,000-m runner competing on a 400-m track crosses the finish line, what is the runner's net displacement? Can this displacement be zero? Explain. A vector has zero magnitude. Is it necessary to specify its direction? Explain. Can a magnitude of a vector be negative? Can the magnitude of a particle's displacement be greater that the distance traveled? If two vectors are equal, what can you say about their components? What can you say about their magnitudes? What can you say about their directions? If three vectors sum up to zero, what geometric condition do they satisfy? A scuba diver makes a slow descent into the depths of the ocean. His vertical position with respect to a boat on the surface changes several times. He makes the first stop 9.0 m from the boat but has a problem with equalizing the pressure, so he ascends 3.0 m and then continues descending for another 12.0 m to the second stop. From there, he ascends 4 m and then descends for 18.0 m, ascends again for 7 m and descends again for 24.0 m, where he makes a stop, waiting for his buddy. Assuming the positive direction up to the surface, express his net vertical displacement vector in terms of the unit vector. What is his distance to the boat? In a tug-of-war game on one campus, 15 students pull on a rope at both ends in an effort to displace the central knot to one side or the other. Two students pull with force 196 N each to the right, four students pull with force 98 N each to the left, five students pull with force 62 N each to the left, three students pull with force 150 N each to the right, and one student pulls with force 250 N to the left. Assuming the positive direction to the right, express the net pull on the knot in terms of the unit vector. How big is the net pull on the knot? In what direction? Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point and what is the compass direction of a line connecting your starting point to your final position? Use a graphical method. For the vectors given in the following figure, use a graphical method to find the following resultants: (a) $$ \overset{\to }{A}+\overset{\to }{B}$$, (b) $$ \overset{\to }{C}+\overset{\to }{B}$$, (c) $$ \overset{\to }{D}+\overset{\to }{F}$$, (d) $$ \overset{\to }{A}-\overset{\to }{B}$$, (e) $$ \overset{\to }{D}-\overset{\to }{F}$$, (f) $$ \overset{\to }{A}+2\overset{\to }{F}$$, (g); and (h) $$ \overset{\to }{A}-4\overset{\to }{D}+2\overset{\to }{F}$$. A delivery man starts at the post office, drives 40 km north, then 20 km west, then 60 km northeast, and finally 50 km north to stop for lunch. Use a graphical method to find his net displacement vector. An adventurous dog strays from home, runs three blocks east, two blocks north, one block east, one block north, and two blocks west. Assuming that each block is about 100 m, how far from home and in what direction is the dog? Use a graphical method. In an attempt to escape a desert island, a castaway builds a raft and sets out to sea. The wind shifts a great deal during the day and he is blown along the following directions: 2.50 km and $$ 45.0\text{°} $$ north of west, then 4.70 km and $$ 60.0\text{°} $$ south of east, then 1.30 km and $$ 25.0\text{°} $$ south of west, then 5.10 km straight east, then 1.70 km and $$ 5.00\text{°} $$ east of north, then 7.20 km and $$ 55.0\text{°} $$ south of west, and finally 2.80 km and $$ 10.0\text{°} $$ north of east. Use a graphical method to find the castaway's final position relative to the island. A small plane flies 40.0 km in a direction $$ 60\text{°} $$ north of east and then flies 30.0 km in a direction $$ 15\text{°} $$ north of east. Use a graphical method to find the total distance the plane covers from the starting point and the direction of the path to the final position. A trapper walks a 5.0-km straight-line distance from his cabin to the lake, as shown in the following figure. Use a graphical method (the parallelogram rule) to determine the trapper's displacement directly to the east and displacement directly to the north that sum up to his resultant displacement vector. If the trapper walked only in directions east and north, zigzagging his way to the lake, how many kilometers would he have to walk to get to the lake? 3.8 km east, 3.2 km north, 7.0 km A surveyor measures the distance across a river that flows straight north by the following method. Starting directly across from a tree on the opposite bank, the surveyor walks 100 m along the river to establish a baseline. She then sights across to the tree and reads that the angle from the baseline to the tree is $$ 35\text{°}$$. How wide is the river? A pedestrian walks 6.0 km east and then 13.0 km north. Use a graphical method to find the pedestrian's resultant displacement and geographic direction. The magnitudes of two displacement vectors are A = 20 m and B = 6 m. What are the largest and the smallest values of the magnitude of the resultant $$ \overset{\to }{R}=\overset{\to }{A}+\overset{\to }{B}?$$